Inference of intrinsic spiking irregularity based on the Kullback-Leibler information

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For a sequence of apparently random neuronal spikes, we wish to determine whether they are derived irregularly in time from a constant rate, or regularly from a fluctuating rate. In our recent paper [1], we proposed an empirical Bayes method of simultaneously characterizing the intrinsic irregularity and the fluctuating rate of an individual spike train.

This method consists of two phases: In the first phase, we estimate the probability distribution of the rate of event occurrence $\lambda(t)$ from an observed sequence of spikes $\{t_i\}_{i=1}^n$, based on the Bayes theorem. For this purpose, we introduce a rate-fluctuating point process, $p_\kappa(\{t_i\}_{i=0}^n|\{\lambda(t)\})$, where the intrinsic irregularity of event occurrence is fixed and parameterized by κ, and a prior probability of a fluctuating rate, $p_\beta(\{\lambda(t)\})$, where the degree of rate fluctuation is specified by β. In the second phase, we search for a set of parameters $\{\hat{\kappa}, \hat{\beta}\}$ which maximize the marginal distribution function,

$$p_{\kappa,\beta}(\{t_i\}_{i=0}^n) = \int p_\kappa(\{t_i\}_{i=0}^n|\{\lambda(t)\}) p_\beta(\{\lambda(t)\}) d\{\lambda(t)\}.$$

Though effective, the empirical Bayes method is based on this *ad hoc* principle of maximizing the marginal likelihood function. Here, we introduce a more basic principle of minimizing the Kullback-Leibler information from the true distribution of spike sequences $q(\{t_i\}_{i=1}^n)$ to a model distribution $p(\{t_i\}_{i=1}^n)$,

$$D(q,p) = -\int q(\{t_i\}_{i=1}^n) \log[p(\{t_i\}_{i=1}^n)/q(\{t_i\}_{i=1}^n)] dt_1 dt_2 \cdots dt_n.$$

Using the hypothetical rate of occurrence $\hat{\lambda}_{\kappa,\beta}(t;\{u_i\}_{i=1}^n)$ obtained by a maximum a posteriori (MAP) estimate from a sample sequence of spikes $\{u_i\}_{i=1}^n$, we construct a model $p(\{t_i\}_{i=1}^n)$. The average over the true distribution $q(\{t_i\}_{i=1}^n)$ is replaced by the numerical average over multiple event sequences.

The Kullback-Leibler information method exhibits a switch of the interpretation of the event sequence between "irregularly derived from a nearly constant rate" to "rather regularly derived from a significantly fluctuating rate". This first-order phase transition is similar to the one exhibited by the empirical Bayes method, not only qualitatively but also quantitatively.

[1] S. Koyama and S. Shinomoto, *Empirical Bayes interpretations of random point events.* preprint.